Zbl 252.10007
Erdős, Paul; Hall, R.R.
On the values of Euler's φ-function. (In English)
Acta Arith. 22, 201-206 (1973). [0065-1036]

Let M denote the set of distinct values of Euler's φ- function, let $m_1, m_2, m_3, \ldots$ be the elements of M arranged as an increasing sequence and let $V(x) = \sum_{m_i \le x} 1$. The authors prove the main result that for each $B > 2\sqrt{2/\log 2}$,

$$V(x) = 0(\pi(x) \exp\{B\sqrt{\log\log x}\})$$

and conjecture that $m_{i+1} - m_i = \omega(\log m_i)$. Let $\omega(n)$ denote the number of prime factors of n counted according to multiplicity, $\omega'(n)$ the number of odd prime factors of n and $\nu(n)$ the number of distinct prime factors of n. By considering the identity

$$(1+y)^{\omega'(n)} = {\sum}'_{d|n} y^{\nu(d)}(1+y)^{\omega(d)-\nu(d)}$$

where $\sum'$ denotes a sum restricted to odd d it is shown that the number of integers $n \le x$ for which $\omega(n) \ge 2\log\log x/\log 2$ is $0(\pi(x)\log\log x)$. From this the main result is proved by dividing the integers $n \le x$ into two special classes and by dividing $V(x)$ into two sums over different subsets of M. An auxiliary result evaluating $\sum_{\omega\{\varphi(m)\}<2\log\log x/\log 2}(1/m)$ is found using complex variable methods.

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Classification:
11A25 Arithmetic functions, etc.
11N37 Asymptotic results on arithmetic functions