Zbl 251.10010
Erdős, Paul
Über die Anzahl der Primfaktoren von $\binom{n}{k}$.
On the number of prime factors of $\binom{n}{k}$. (In German)
Arch. der Math. 24, 53-56 (1973).

Let $V(m)$ denote the number of different prime factors of m. *H.Scheid* [Arch. Math. 20, 581-582 (1969; Zbl 195.33001)] proved that for $2 < 2k \leq n$

$$V\binom{n}{k} > \frac{k\log 2}{\log 2k}.$$

The author here proves the following theorem: For every $\epsilon > 0$ and $k > k_0(\epsilon)$, and for $n \geq 2k$,

$$V\binom{n}{k} > (1-\epsilon)\frac{k\log 4}{\log k}.$$

To show that the above result is in a sense accurate, he further proves $V\binom{2k}{k} < (1+\epsilon)\frac{k\log 4}{\log k}$. An analogous proof is stated to hold for

$$V\binom{n}{k} < (1+\epsilon)\frac{n\log 2}{\log n}.$$

Scheid considered it probable that, for fixed k, $V\binom{n}{k}$ does not tend to infinity. The author in fact proves this statement to be true. If $n > 2\cdot k!$, he also proves that $V\binom{n}{k} \geq k$, and finally states the following conjecture. For almost all $n < k^{1+\alpha}$

$$V\binom{n}{k} = (1+0(1))k\log(1+\alpha).$$

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Classification:
11A41 Elemementary prime number theory
05A10 Combinatorial functions