
Zbl 174.01804

Erdős, Pál; Hajnal, András; Milner, E.C.

On sets of almost disjoint subsets of a set (In English)

Acta Math. Acad. Sci. Hung. 19, 209-218 (1968). [0001-5954]

Two sets are said to be almost disjoint if the cardinality of their intersection is strictly less than the cardinality of either. A transversal of disjoint nonempty sets is a set contained in their union which has one element in common with each set. Sierpinski showed that m disjoint sets each of power m posses more than m almost disjoint transversals. In this paper a number of related results are presented. These include: 1. $\aleph_{\alpha+1}$ disjoint sets of power $\aleph_\alpha$ possess a maximal set of $\aleph_{\alpha+1}$ almost disjoint transversals. (Every other transversals being not almost disjoint from one of them.) 2. $\aleph_{\alpha+1}$ disjoint sets of power $\aleph_\alpha$ possess a set of transversals whose intersection has cardinality strictly less than $\aleph_\alpha$. 3. If the cofinality cardinal of m is $\aleph_0$ and if $n < m$ implies $2^n < m$, then there is no maximal set of power m of almost disjoint transversals of $\aleph_0$ disjoint sets of power $\aleph_0$.

Several other related results, all concerned with maximality and cardinality of sets of transversals are also presented, as in a generalization of a result of F. C. Cater itself extending Sierpinski's theorem.

D.Kleitman

Classification:
05D15 Transversal (matching) theory
04A20 Combinatorial set theory