
Zbl 164.47502
Darling, D.A.; Erdős, Pál
On the recurrence of a certain chain (In English)
Proc. Am. Math. Soc. 19, 336-338 (1968). [0002-9939]

Let balls be placed successively and independently in urns $U_1, U_2, ...$, urn U_1 receiving each ball with probability p_i, $i = 1, 2, ...$ After n balls have been placed let L_h be the number of urns containing an odd number of balls. The event ($L_h = 0$ for infinitely many n) has probability one or zero, termed respectively the "recurrent" and the "transient" cases. In *F. Spitzer*, Principles of random walk (1964; Zbl 119.34304), p. 94, it was stated that "it seems impossible to obtain a general criterion in terms of $\{p_k\}$ to ensure the recurrent case", and by *D. A. Darling* [Proc. 5th Berkeley Sympos. Math. Stat. Probab., Univ. Calif. 1965/66, 2, No. 1, 345-350 (1967; Zbl 201.50801)] it was stated "it would appear that the necessary and sufficient conditions are rather delicate and not to be exhibited in neat form".

In this note we clarify matters, showing that the condition (1) given below, previously known to be sufficient for recurrence is also necessary. Without loss of generality we assume $p_i > 0$, $i = 1, 2, ..., p_1 \geq p_2 \geq p_3 \geq ...$, and we set $f_n = p_n + p_{n+1} + ...$, so that $f_1 = 1$ and f_n decreases monotonically to zero. Theorem. A necessary and sufficient condition for recurrence is that (1) $\sum_1^\infty \frac{1}{2^n f_n} = \infty$.

Classification:
60-99 Probability theory and stochastic processes