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Erdős, Pál; Ulam, S.

On equations with sets as unknowns (In English)

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The authors prove among others the following theorems: Let $|S| \geq \aleph_0$, $2 \leq k_1 \leq k_2 \leq ..., k_n \to \infty$ and $S = \bigcup_{l=1}^{k_n} A_l^{(n)}$, $n = 1, 2, ...$ be a decomposition of S into k_n disjoint sets. Then there is always an $l_n, 1 \leq l_n \leq k_n$ so that $\left| S - \bigcup_{n=l}^{\infty} A_{l_n}^{(n)} \right| \geq \aleph_0$. Assume $2^{\aleph_0} = \aleph_1$, $|S| = \aleph_1$. Then S can be decomposed in $\aleph_1$ ways as the union of disjoint sets $S = \bigcup_{1 \leq \alpha < \omega_1} A_\alpha^{(\beta)}$, $1 \leq \beta < \omega_1$ so that if we choose any one of the sets $A_{\alpha_1}^{(\beta_1)}$ for $\aleph_0$ different β_1 then $\left| S - \bigcup_{l=1}^{\infty} A_{\alpha_1}^{(\beta_1)} \right| \leq \aleph_0$. Several extensions and generalizations are discussed and many unsolved problems and relations with other problems and results in set theory are discussed.

Classification:
05D05 Extremal set theory
04A20 Combinatorial set theory