Zbl 086.34001
Erdős, Paul; Rényi, Alfréd
On the central limit theorem for samples from a finite population. (In English. RU summary)
Publ. Math. Inst. Hung. Acad. Sci. 4, 49-61 (1959).

Let $a_1, ..., a_n$ be arbitrary real numbers. Let us consider all possible $\binom{n}{s}$ sums $\sum_{k=1}^{s} a_{i_k}$, $1 \leq i_1 < \cdots < i_s \leq n$ formed by choosing s arbitrary different elements of the sequence $a_1, a_2, ..., a_n$. Let us put

$$M_n = \sum_{k=1}^{n} a_k,$$

$$D_n = \left\{ \sum_{k=1}^{\infty} \left(a_k - \frac{M_n}{n} \right)^2 \right\}^{1/2},$$

$$D_{n,s} = D_n \left\{ \frac{s}{n} \left(1 - \frac{s}{n} \right) \right\}^{1/2}.$$

Let $N_{n,s}(x)$ denote the number of those sums $a_{i_1} + \cdots + a_{i_s}$ which don't exceed $\left(\frac{s}{n}\right) M_n + xD_{n,s}$ and put $F_{n,s}(x) = N_{n,s}(x)/\binom{n}{s}$.
In the paper the authors ask about conditions concerning the sequence $\{a_n\}$ and s under which

$$F_{n,s}(x)_{(n)} \to \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\tau^2/2}\, d\tau.$$

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Classification:
60F05 Weak limit theorems