Zbl 053.08002
Erdős, Pál; Straus, E.G.
On linear independence of sequences in a Banach space. (In English)
Pac. J. Math. 3, 689-694 (1953). [0030-8730]

This paper gives an answer to a problem raised by *A.Dvoretzky*: given a sequence of (algebraically) linearly independent unit vectors of a Banach space, does there exists a subsequence linearly independent in some stronger sense? The authors prove that, given any positively valued function $\varphi(n)$, every sequence of linearly independent unit vectors of a Banach space contains a subsequence x_n independent in the following sense: if C_n^k are scalars such that

$$(1)\ \sup_k |C_n^k| < \varphi(n), \qquad (2)\ \lim_{k\to\infty} \sum_{n=1}^{\infty} C_n^k x_n = 0,$$

then $\lim_{k\to\infty} C_n^k = 0$ for $n = 1, 2, ...$ This implies a fortiori that these vectors are linearly independent in the following sense: if $\sum_{n=1}^{\infty} c_n x_n = 0$, then $c_n = 0$ for $n = 1, 2, ...$ The authors prove also that if the condition (1) is dropped in the definition of linear independence, their theorem is no longer true.
[The following misprints are to be noted: p. 689, line 13 the inequality is to be read $|C_n^{(k)}| < \varphi(n)$; p. 690, line 16 replace x_{n_i} by x_n; p. 691, line 20 replace Q by O.]

A.Alexiewicz

Classification:
46B99 Normed linear spaces and Banach spaces