Zbl 023.02201

Erdős, Paul; Turán, Paul

On the uniformly-dense distribution of certain sequences of points. (In English)

Ann. of Math., II. Ser. 41, 162-173 (1940).

Le théorème suivant est démontré: soient $\varphi_1^{(n)}, \varphi_2^{(n)}, ..., \varphi_n^{(n)}$, $n = 1, 2, 3, ...$, des nombres satisfaisant aux conditions $0 \leq \varphi_1^{(n)} < \varphi_2^{(n)} < \cdots < \varphi_n^{(n)} \leq \pi$. Si les polynômes $\omega_n(z) = \prod_{\nu=1}^n (z - \cos\varphi_\nu^{(n)})$ satisfont à l'inégalité $|\omega_n(z)| \leq 2^{-n} A(n)$, où $A(n)$ est une fonction croissante tendant vers l'infini, alors pour chaque intervalle (α, β), $0 \leq \alpha < \beta \leq \pi$ on a

$$\left| \sum\nolimits_{\nu}{}_{\alpha \leq \varphi_\nu^{(n)} \leq \beta} \sum\nolimits_{\nu} 1 - \frac{\beta - \alpha}{\pi} n \right| < \frac{8}{\log 3} (n \log A(n))^{\frac{1}{2}}.$$

The following theorem is proved: Let $\varphi_1^{(n)}, \varphi_2^{(n)}, ..., \varphi_n^{(n)}$, $n = 1, 2, 3, ...$, be numbers satisfying the conditions $0 \leq \varphi_1^{(n)} < \varphi_2^{(n)} < \cdots < \varphi_n^{(n)} \leq \pi$. If the polynomials $\omega_n(z) = \prod_{\nu=1}^n (z - \cos\varphi_\nu^{(n)})$ satisfy the inequality $|\omega_n(z)| \leq 2^{-n} A(n)$, where $A(n)$ is an increasing function tending to infinity, then for each interval (α, β), $0 \leq \alpha < \beta \leq \pi$ the following inequality is valid

$$\left| \sum\nolimits_{\nu}{}_{\alpha \leq \varphi_\nu^{(n)} \leq \beta} \sum\nolimits_{\nu} 1 - \frac{\beta - \alpha}{\pi} n \right| < \frac{8}{\log 3} (n \log A(n))^{\frac{1}{2}}.$$

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