Zbl 012.01101
Erdős, Paul
Note on consecutive abundant numbers. (In English)
J. London Math. Soc. 10, 128-131 (1935).

Continuing his work on abundant numbers (Zbl 010.10303; Zbl 010.39103), the author proves that there are two absolute constants c_1, c_2 such that for all large n there are at least $c_1 \log\log\log n$ but not more than $c_2 \log\log\log n$ consecutive abundant numbers less than n. The first result is obtained by taking $a_1 = 2 \cdot 3,\ a_2 = 5 \cdot 7 ... p_1, a_3 = p_2 ... p_3, ...,$ where p_1 is the least prime making a_2 abundant, p_2 the next prime, p_3 the least prime making a_3 abundant, and so on, and solving the congruences $x \equiv r-1 (\mathrm{mod} a_r)$, $r = 1, 2, ..., \nu$. The second result is proved by considering those numbers $b_1, ..., b_2$, of a set of k consecutive abundant numbers, which are not divisible by any prime less than a particular fixed prime q. We have

$$2^z \le \prod_{i=1}^{z} \frac{\sigma(b_i)}{b_i} < \prod_{\substack{p>q \\ p|b_1,\dots b_z}} (\frac{p}{p-1})^{[\frac{k}{p}]+1},$$

and

$$z > k \prod_{p<q} \left(1 - \frac{1}{p}\right) - 2^q,$$

the latter by the sieve of Eratosthenes. From these inequalities the upper bound for k is deduced.

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Classification:
11A25 Arithmetic functions, etc.